Algebra II — Lesson 11 - Cartesian Plane

Dr. Paul L. Bailey — Monday, August 29, 2022

1. Historical Background

Synthetic geometry (sometimes referred to as *axiomatic geometry* or even *pure geometry*) is the study of geometry without the use of coordinates or formulae. It relies on the axiomatic method and the tools directly related to them, that is, compass and straightedge, to draw conclusions and solve problems.

The world's knowledge of synthetic geometry was summarized by Euclid in his thirteen book series, *The Elements*, circa 300 B.C. The ancient greeks had no algebra at all; there mathematics related primarily to geometry, and was written in words, not symbols.

Analytic geometry uses a coordinate system to translate geometric ideas into algebraic expressions and equations. Only after the introduction of coordinate methods almost two thousand years after Euclid was there a reason to introduce the term "synthetic geometry" to distinguish this approach to geometry from the algebraic and analytic approaches.

Cartesian coordinates were first introduced to the study of a plane the 17th century. The adjective Cartesian refers to the French mathematician and philosopher René Descartes, who published this idea in 1637 while he was resident in the Netherlands. It was independently discovered by Pierre de Fermat, who also worked in three dimensions, although Fermat did not publish the discovery.

According to Felix Klein, "synthetic geometry is that which studies figures as such, without recourse to formulae, whereas analytic geometry consistently makes use of such formulae as can be written down after the adoption of an appropriate system of coordinates."

Some of the summary above was borrowed from Wikipedia.

2. The Real Line

We have discussed how there exists a one-to-one correspondence between points on a line and decimal expansions. For this reason, we defined real numbers as decimal expansions.

The set of all real numbers is denoted $\mathbb{R}$. Geometrically, we view $\mathbb{R}$ as a line.

3. The Cartesian Plane

An *ordered pair* consists of two elements in a specific order. The ordered pair which has element a first and element b second is denoted (a, b).

Ordered pairs have the defining property

$$(a, b) = (c, d) \quad \text{if and only if} \quad a = b \text{ and } c = d.$$

The set of all ordered pairs of real numbers is denoted $\mathbb{R}^2$. That is,

$$\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}.$$

Geometrically, we view $\mathbb{R}^2$ as a plane.

As you have seen in previous courses, take two lines (copies of $\mathbb{R}$) and place them at right angles to obtain the *cartesian coordinate system.* This is a plane on which every point is labeled with an ordered pair of real numbers. The horizontal line is usually the x-axis, and the vertical line is usually the y-axis.

4. Plotting Points in $\mathbb{R}^2$

When you plot a few points, it is not necessary to be overly accurate; you just have to estimate the distances along the axes for the points you want to plot. For example, this is how one could draw the following points:

$$(0,0),(1,3),(5,7),(-1,5),(3,-5),(-7,-2)$$